Math 214 – Foundations of Mathematics
Homework 3

Your name

Due noon Sept 21, 2012

Solve the following problems. Please remember to use complete sentences and good grammar.

1. *(4 Points)* For the following, state whether they are true or not. Then, prove your answer.
   (a) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, xy = 1$;
   (b) $\exists n \in \mathbb{N}, \exists m \in (\mathbb{N} - \{1\}), nm = 1$.
2. *(4 Points)* Prove that if x and y are positive real numbers, then $\sqrt{x+y} \neq \sqrt{x} + \sqrt{y}$.
3. *(4 Points)* Prove that the product of an irrational number and a nonzero rational number is irrational.
4. *(4 Points)* Recall that for a given $S \subseteq \mathbb{R}$, the maximum element of S (written $\max\{n : n \in S\}$) as the number $\alpha \in S$ such that for all $\beta \in S$, $\alpha \geq \beta$.
   Let $A = \{n \in \mathbb{N} : \sqrt{n} \notin \mathbb{Q}\}$. Show that $\max\{n : n \in A\}$ does not exist.
5. *(4 Points)* Use induction to prove that, for all $n \in \mathbb{N}$,
$$\sum_{k=0}^{n} (2k+1) = (n+1)^2.$$
6. *(4 Points)* Use induction to prove that for all integers $n \geq 3$, $n^3 \leq 3^n$.