Math 214 – Foundations of Mathematics
Homework 6

Due Oct 19, 2012

Your name

Solve the following problems. Show all your work.

1. Let a, b, c be integers. Prove that if $3|(abc - 1)$, then $3|(a - 1)$, $3|(b - 1)$, or $3|(c - 1)$.

2. Let $d = gcd(a, b)$. If $a = da'$ and $b = db'$, show that $gcd(a', b') = 1$.

3. Let $a, b \in \mathbb{Z}$, where not both a and b are 0. Show that there are infinitely many pairs x, y of integers such that $gcd(a, b) = ax + by$.

4. Show that $n + 1$ and $3n + 2$ are coprime.

5. Prove that $\sqrt[3]{3}$ and $\log_{10} 234$ are irrational numbers.

6. For integers a and b, let $lcm(a, b)$ be the least positive multiplier of a and b. Show that $lcm(a, b) \cdot gcd(a, b) = ab$. (hint: express gcd and lcm in terms of the prime factors of a and b. you may need to prove that $\max\{m, n\} + \min\{m, n\} = m + n$ first.)