Math 214 – Foundations of Mathematics
Homework 7

Due Oct 26, 2012

Your name

Solve the following problems. Show all your work. Four points each if not specified.

1. Find the last digit of 7^{2011}.
2. Without using induction, prove that $(10^{n+1} - 9n - 10) \equiv 0 \pmod{81}$ for every nonnegative integer n. (Remark: this is the same problem, in different form, in a previous homework set. But here you are not allowed to use induction.)
3. (6 points) Solve the following problems in $\mathbb{Z}_n$.
   (a) In $\mathbb{Z}_8$, express the following sums and products as $[r]$, where $0 \leq r < 8$:
   $$[3] + [6], [3][6], [-13] + [138], [-13][138]$$
   (b) Let $[a], [b] \in \mathbb{Z}_8$. If $[a][b] = [0]$, does it follow that $[a] = [0]$ or $[b] = [0]$?
   (c) Prove that for any prime p, if $[a], [b] \in \mathbb{Z}_p$, then $[a][b] = [0]$ implies $[a] = [0]$ or $[b] = [0]$.
4. Prove that the multiplication in $\mathbb{Z}_n$, $n \geq 2$, defined by $[a][b] = [ab]$ is well-defined.
5. A relation R is defined on $\mathbb{Z}$ by $(a, b) \in R$ if $|a - b| \leq 2$. Which of the properties reflexive, symmetric, and transitive does the relation R possess? Justify your answers.
6. Let R be a relation defined on $Z - \{0\}$ by $(a, b) \in R$ if $ab > 0$. Show that R is an equivalence relation on $Z - \{0\}$.