Two examples in Mathematical Induction

Once winner, always winner??

September 20, 2010

Example:

- Proposition: For all positive integers n, the number $n^2 + n$ is even.
- Proof. we use induction on n.
  1. When $n = 1$: $1^2 + 1 = 2$ is even. True.
  2. Suppose that it is true when $n = k$, that is $k^2 + k$ is even. We need to show that it is true for $n = k + 1$, that is $(k+1)^2 + (k+1)$ is even.
     Since $(k+1)^2 + (k+1) = (k+1)(k+1+1)$ is the product of two consecutive numbers and one of them must be even, $(k+1)^2 + (k+1)$ is even.
  3. By mathematical induction, the number $n^2 + n$ is even for all positive integers n.

  Is this a math induction proof?

All people have the same sex (!?)

Proof. Let $P(n)$ be the statement that "in any set of n people, all members of the set are the same sex".

If we have a set consisting of one person, then clearly all the members of the set are of the same sex, so $P(1)$ is true.

Suppose that $P(k)$ is true. Then in any set of k people, all the members of the set are of the same sex. In order to show that $P(k+1)$ is true, we need to show that in any set of $k+1$ people, all the members of the set are of the same sex.

Take a set of $k+1$ people. Call these people $a_1, a_2, \ldots, a_k, a_{k+1}$. If we send one person out of the room, say a_1 , then we have a set of k people left in the room, so by induction hypothesis they are all of the same sex. Now bring a_1 back into the room and set a_2 out. Again there is a set of k people left in the room, so by the assumption that they are all of the same sex. Now observe that everyone in the original set of $k+1$ people is of the same sex as a_3, so the are all of the same sex. By the Principle of Mathematical Induction, $P(n)$ is true.