

Math 412 Homework 1

your name

Due date: 11:59pm Sept 2, 2016

Solve the following problems. Please remember to use complete sentences and good grammar. Each problem is 4 points.

1. Show that if $2|n$ and $3|n$, then $6|n$.
2. Show that the square of every odd integer is of the form $8k + 1$.
3. Show that $11|a^2 + 5b^2$ if and only if $11|a$ and $11|b$ for $a, b \in \mathbb{Z}$.
4. Show that $8a + 3$ and $5a + 2$ are relatively prime for all integers a .
5. Show that $(ac, bc) = |c|(a, b)$ for any integers a, b, c .
6. For the following linear diophantine equation, either find all solutions or show that there is no integral solutions:
$$25x + 95y = 970$$
7. (Bonus) Suppose that the coefficients of the polynomial $f(x)$ are integers. If $f(a) = f(a+2) = (a+1)^2$ for some positive integer a and $0 < f(0) < (a+1)^2$, find the value of $f(0)$.