Math 412 Homework 12

your name

Due date: noon Dec 4, 2015

Solve the following problems. Please remember to use complete sentences and good grammar.

1. Show that $1+i$ divides a Gaussian integer $a+ib$ if and only if a and b are both even or both odd.
2. Find all Gaussian primes of the form α^2+1, where α is a Gaussian integer.
3. Let $\alpha, \beta \in \mathbb{Z}[i]$ and are coprime. If $\alpha\beta = \gamma^2$ for some $\gamma \in \mathbb{Z}[i]$, then α and β can also be written as square, up to a factor of units.
4. Find an inverse of 4 modulo $5+2i$ and solve the linear congruence in Gaussian integers
$$4x \equiv -3+4i \pmod{5+2i}$$
5. show that if x and y are integers such that $x^2+1 = y^3$, then $x-i$ and $x+i$ are relatively prime.
6. Use the Gaussian integers to find all solutions to the diophantine equation $x^2+y^2 = z^3$ in rational integers x, y and z.