

# Math 412 Homework 3

your name

Due date: Sept 18, 2015

Solve the following problems. Please remember to use complete sentences and good grammar. Each problem is 4 points.

1. show that  $\frac{(2n)!}{(n!)^2}$  is even.
2. Show that  $x^2 - 2y^2 = 77$  has no integer solution.
3. Solve the following system of linear congruences:  $5x \equiv 1 \pmod{9}$ ,  $x \equiv 8 \pmod{15}$ ,  $x \equiv 3 \pmod{25}$ .
4. Show that the system of congruences  $x \equiv a_1 \pmod{m_1}$ ,  $x \equiv a_2 \pmod{m_2}$  has a solution if and only if  $(m_1, m_2) | (a_1 - a_2)$ . Show that when there is a solution, it is unique modulo  $[m_1, m_2]$ .
5. Solve the equation  $x^2 + 5x + 13 \equiv 0 \pmod{3^4}$ .
6. Show that  $n, n + 2$  are both prime if and only if

$$4((n-1)! + 1) \equiv -n \pmod{n(n+2)}.$$