1. Let $A = \begin{bmatrix} 1 & 3 & -4 & 7 \\ 2 & 6 & 5 & 1 \\ 3 & 9 & 4 & 5 \end{bmatrix}$.

   (a) Find all the solutions of the non-homogeneous system $Ax = b$, and write them in parametric form, where $b = \begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix}$.

   (b) Find all the solutions of the homogeneous system $Ax = 0$, and write them in parametric form.

   (c) Are the columns of the matrix A linearly independent? Write down a linear relation between the columns of A if they are dependent.

2. Let $S = Span\{u_1, u_2, u_3, u_4\}$. where

$$u_1 = \begin{bmatrix} 1 \\ -2 \\ 3 \\ 1 \end{bmatrix}, u_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ -2 \end{bmatrix}, u_3 = \begin{bmatrix} 1 \\ -3 \\ 2 \\ 3 \end{bmatrix}, u_4 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ -3 \end{bmatrix}$$

(a) Are u_1, u_2, u_3, u_4 linearly independent?

(b) Find all the vectors $u = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$ such that the u is in S. Write these u in parametric form. Justify your answer.

(c) Is $v = \begin{bmatrix} -1 \\ 3 \\ -2 \\ 1 \end{bmatrix}$ in S? Is $w = \begin{bmatrix} 1 \\ 3 \\ -2 \\ 1 \end{bmatrix}$ in S?

3. Consider the 4×4 matrix:

$$A = \begin{bmatrix} \lambda & 1 & 0 & 0 \\ 1 & \lambda & 1 & 0 \\ 0 & 1 & \lambda & 1 \\ 0 & 0 & 1 & \lambda \end{bmatrix}$$

(a) Find $det(A)$;

(b) Find A^{-1};

(c) find LU-decomposition of A.

4. Let $e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ and $e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$. Suppose $T : R^3 \mapsto R^2$ is a linear transformation such that $T(e_1 + e_2) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $T(e_1 - e_2) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $T(e_1 + e_2 + e_3) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$. What is $T(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix})$?

5. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation defined by

$$T(x) = (x_1 - 2x_2, x_1 + 5x_2).$$

(a) Determine the standard matrix, A, of T.

(b) Find A^{-1}.

(c) Is T is *one to one*? *onto*? Why?

(d) If $Ax = \begin{bmatrix} 14 \\ 7 \end{bmatrix}$, solve for x.