CHAPTER 1 SUPPLEMENTARY EXER

1. Mark each statement True or False. Justify each answer. (If true, cite appropriate facts or theorems. If false, explain why or give a counterexample that shows why the statement is not true in every case.

a. Every matrix is row equivalent to a unique matrix in echelon form.

b. Any system of n linear equations in n variables has at most n solutions.

c. If a system of linear equations has two different solutions, it must have infinitely many solutions.

d. If a system of linear equations has no free variables, then it has a unique solution.

e. If an augmented matrix $[\,A \quad \mathbf{b}\,]$ is transformed into $[\,C \quad \mathbf{d}\,]$ by elementary row operations, then the equations $A\mathbf{x} = \mathbf{b}$ and $C\mathbf{x} = \mathbf{d}$ have exactly the same solution sets.

f. If a system $A\mathbf{x} = \mathbf{b}$ has more than one solution, then so does the system $A\mathbf{x} = \mathbf{0}$.

g. If A is an $m \times n$ matrix and the equation $A\mathbf{x} = \mathbf{b}$ is consistent for some $\mathbf{b}$, then the columns of A span $\mathbb{R}^m$.

h. If an augmented matrix $[\,A \quad \mathbf{b}\,]$ can be transformed by elementary row operations into reduced echelon form, then the equation $A\mathbf{x} = \mathbf{b}$ is consistent.

i. If matrices A and B are row equivalent, they have the same reduced echelon form.

j. The equation $A\mathbf{x} = \mathbf{0}$ has the trivial solution if and only if there are no free variables.

k. If A is an $m \times n$ matrix and the equation $A\mathbf{x} = \mathbf{b}$ is consistent for every $\mathbf{b}$ in $\mathbb{R}^m$, then A has m pivot columns.

l. If an $m \times n$ matrix A has a pivot position in every row, then the equation $A\mathbf{x} = \mathbf{b}$ has a unique solution for each $\mathbf{b}$ in $\mathbb{R}^m$.

m. If an $n \times n$ matrix A has n pivot positions, then the reduced echelon form of A is the $n \times n$ identity matrix.

n. If 3×3 matrices A and B each have three pivot positions, then A can be transformed into B by elementary row operations.

o. If A is an $m \times n$ matrix, if the equation $A\mathbf{x} = \mathbf{b}$ has at least two different solutions, and if the equation $A\mathbf{x} = \mathbf{c}$ is consistent, then the equation $A\mathbf{x} = \mathbf{c}$ has many solutions.

p. If A and B are row equivalent $m \times n$ matrices and if the columns of A span $\mathbb{R}^m$, then so do the columns of B.

q. If none of the vectors in the set $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ in $\mathbb{R}^3$ is a multiple of one of the other vectors, then S is linearly independent.

r. If $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent, then $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are not in $\mathbb{R}^2$.

s. In some cases, it is possible for four vectors to span $\mathbb{R}^5$.

t. If $\mathbf{u}$ and $\mathbf{v}$ are in $\mathbb{R}^m$, then $-\mathbf{u}$ is in Span$\{\mathbf{u}, \mathbf{v}\}$.

u. If $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are nonzero vectors in $\mathbb{R}^2$, then $\mathbf{w}$ is a linear combination of $\mathbf{u}$ and $\mathbf{v}$.

v. If $\mathbf{w}$ is a linear combination of $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^n$, then $\mathbf{u}$ is a linear combination of $\mathbf{v}$ and $\mathbf{w}$.

w. Suppose that $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ are in $\mathbb{R}^5$, $\mathbf{v}_2$ is not a multiple of $\mathbf{v}_1$, and $\mathbf{v}_3$ is not a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$. Then $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly independent.

x. A linear transformation is a function.

y. If A is a 6×5 matrix, the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ cannot map $\mathbb{R}^5$ onto $\mathbb{R}^6$.

z. If A is an $m \times n$ matrix with m pivot columns, then the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ is a one-to-one mapping.

CHAPTER 2 SUPPLEMENTARY EX

1. Assume that the matrices mentioned in the statements below have appropriate sizes. Mark each statement True or False. Justify each answer.

a. If A and B are $m \times n$, then both AB^T and A^TB are defined.

b. If $AB = C$ and C has 2 columns, then A has 2 columns.

c. Left-multiplying a matrix B by a diagonal matrix A, with nonzero entries on the diagonal, scales the rows of B.

d. If $BC = BD$, then $C = D$.

e. If $AC = 0$, then either $A = 0$ or $C = 0$.

f. If A and B are $n \times n$, then $(A + B)(A - B) = A^2 - B^2$.

g. An elementary $n \times n$ matrix has either n or $n + 1$ nonzero entries.

h. The transpose of an elementary matrix is an elementary matrix.

i. An elementary matrix must be square.

j. Every square matrix is a product of elementary matrices.

k. If A is a 3×3 matrix with three pivot positions, there exist elementary matrices $E_1, \ldots, E_p$ such that $E_p \cdots E_1 A = I$.

l. If $AB = I$, then A is invertible.

m. If A and B are square and invertible, then AB is invertible, and $(AB)^{-1} = A^{-1}B^{-1}$.

n. If $AB = BA$ and if A is invertible, then $A^{-1}B = BA^{-1}$.

o. If A is invertible and if $r \neq 0$, then $(rA)^{-1} = rA^{-1}$.

p. If A is a 3×3 matrix and the equation $A\mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has a unique solution, then A is invertible.

CHAPTER 3 SUPPLEMENTARY EXE

1. Mark each statement True or False. Justify each answer. Assume that all matrices here are square.

   a. If A is a 2×2 matrix with a zero determinant, then one column of A is a multiple of the other.

   b. If two rows of a 3×3 matrix A are the same, then $\det A = 0$.

   c. If A is a 3×3 matrix, then $\det 5A = 5 \det A$.

   d. If A and B are $n \times n$ matrices, with $\det A = 2$ and $\det B = 3$, then $\det(A + B) = 5$.

   e. If A is $n \times n$ and $\det A = 2$, then $\det A^3 = 6$.

   f. If B is produced by interchanging two rows of A, then $\det B = \det A$.

   g. If B is produced by multiplying row 3 of A by 5, then $\det B = 5 \cdot \det A$.

   h. If B is formed by adding to one row of A a linear combination of the other rows, then $\det B = \det A$.

   i. $\det A^T = -\det A$.

   j. $\det(-A) = -\det A$.

   k. $\det A^T A \geq 0$.

   l. Any system of n linear equations in n variables can be solved by Cramer's rule.

   m. If $\mathbf{u}$ and $\mathbf{v}$ are in $\mathbb{R}^2$ and $\det [\mathbf{u} \ \ \mathbf{v}] = 10$, then the area of the triangle in the plane with vertices at $\mathbf{0}$, $\mathbf{u}$, and $\mathbf{v}$ is 10.

   n. If $A^3 = 0$, then $\det A = 0$.

   o. If A is invertible, then $\det A^{-1} = \det A$.

   p. If A is invertible, then $(\det A)(\det A^{-1}) = 1$.