Math 412 Homework 11

your name

Due date: Nov 13, 2015

Solve the following problems. Please remember to use complete sentences and good grammar.

1. Show that if $a_0 > 0$, then $p_k/p_{k-1} = [a_k, a_{k-1}, \ldots, a_1, a_0]$ and $q_k/q_{k-1} = [a_k, a_{k-1}, \ldots, a_2, a_1]$, where $C_{k-1} = p_{k-1}/q_{k-1}$ and $C_k = p_k/q_k$, $k \geq 1$, are successive convergence of the continued fraction $[a_0, a_1, \ldots, a_n]$.

2. Let $\alpha > 1$ be an irrational number. Show that the k-th convergent of the simple continued fraction of $1/\alpha$ is the reciprocal of the $(k-1)$-th convergent of the simple continued fraction of α.

3. Let α be an irrational number and let p_j/q_j be the j-th convergent of the simple continued fraction expansion of α. Show that at least one of any three consecutive convergence satisfies the inequality $|\alpha - p_j/q_j| < 1/(\sqrt{5}q_j^2)$. Conclude that there are infinitely many rational numbers p/q, where p and q are integers with $q \neq 0$, such that $|\alpha - p/q| < 1/(\sqrt{5}q^2)$.

4. Show that the simple continued fraction $\sqrt{d}$, where d is a positive integer, has period length one if and only if $d = a^2 + 1$, where a is a nonnegative integer.

5. Show that if p_k/q_k is a convergent of the simple continued fraction expansion of $\sqrt{d}$, then $|p_k^2 - dq_k^2| < 1 + 2\sqrt{d}$.

6. (Bonus) Let k be a positive integer and $D_k = (3^k + 1)^2 + 3$. Show that the simple continued fraction of $\sqrt{D_k}$ has a period of length $6k$.