Math 412 Homework 2

your name

Due date: Sept 11, 2015

Solve the following problems. Please remember to use complete sentences and good grammar. Each problem is 4 points.

1. Show that if p_k is the k-th prime, where k is a positive integer, then $p_n \leq p_1 p_2 \ldots p_{n-1} + 1$ for all integers n with $n \geq 3$.

2. Show that the integer $Q_n = n! + 1$, where n is a positive integer, has a prime divisor greater than n. Conclude that there are infinitely many primes.

3. Show that there are infinite many primes of the form $3k - 1$.

4. Let $S = \{\log_{10} p : p \text{ prime}\}$. Prove that the elements of S are linearly independent over Q, the rationals.

5. Show that $\sqrt{3} - \sqrt[3]{2}$ and $\log_{10} 5432$ are irrational.

6. Find the prime factorization of 32!. How many zeros are at the end of 32!?

7. Find the last two digits of 9^{9^9}.

8. (extra credit) Show that for any integer $n \geq 2$, $1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}$ is not an integer.

9. (extra credit) Show that $\sin \frac{\pi}{7}$ is irrational.