# Math 412 Homework 4

your name

Due date: Sept 25, 2015

Solve the following problems. Please remember to use complete sentences and good grammar. Four points each.

1. Find the least nonnegative remainder of 30! (mod 899).
2. let $r_1, r_2, \ldots, r_{\phi(m)}$ be a reduced system of residues modulo $m = 2^l$, where $l \geq 3$, then
$$\prod_i r_i \equiv 1 \pmod{2^l}.$$
3. let $r_1, r_2, \ldots, r_{\phi(m)}$ be a reduced system of residues modulo $m = 2p^l$, where $l \geq 1$ and $p$ is an odd prime, then $\prod_i r_i \equiv -1 \pmod{2p^l}$.
4. Show that if $n$ is an odd positive integer or if $n$ is a positive divisible by 4, then
$$1^3 + 2^3 + \ldots + (n-1)^3 \equiv 0 \pmod{n}.$$
5. Let $(m, n) = 1$. Show that $m^{\phi(n)} + n^{\phi(m)} \equiv 1 \pmod{mn}$.
6. Find the last two digits of $a_{1001}$ if $a_1 = 7, a_n = 7^{a_{n-1}}$.
7. (Extra credit problem) Let $m \geq 3$, $r_1, r_2, \ldots, r_m$ and $r'_1, r'_2, \ldots, r'_m$ are two complete system of residues modulo $m$. Show that $r_1r'_1, r_2r'_2, \ldots, r_mr'_m$ is a not a complete system of residues modulo $m$.