

Math 412 Homework 6

your name

Due date: Oct 9, 2015

Solve the following problems. Please remember to use complete sentences and good grammar.

1. (4 points) Show that if n is a positive integer, then $(\sum_{d|n} \tau(d))^2 = \sum_{d|n} \tau(d)^3$.
2. (4 points) Show that if $f(n) = \sum_{d|n} \mu(d)F(n/d)$, then $F(n) = \sum_{d|n} f(d)$.
3. (4 points) The Dirichlet product of two arithmetic functions f and g is defined to $(f * g)(n) = \sum_{d|n} f(d)g(n/d)$. Show that if f and g are multiplicative functions, then $f * g$ is also multiplicative.
4. (4 points) Show that $\sum_{d|n} \mu^2(d) = 2^{\omega(n)}$, where $\omega(n)$ is the number of distinct prime factors of n .
5. (4 points) Show that if n is a positive integer, then $\phi(n) = n \sum_{d|n} \mu(d)/d$.
6. A positive integer n is called super perfect if $\sigma(\sigma(n)) = 2n$. Show that
 - (a) (2 points) if $n = 2^q$, where $2^{q+1} - 1$ is prime, then n is superperfect.
 - (b) (4 points) every even superperfect number is of form $n = 2^q$, where $2^{q+1} - 1$ is prime.
7. (Bonus, 4 points) Show that if $n = p^2$, where p is an odd prime, then n is not superperfect.