Math 412 Homework 8

your name

Due date: Oct 30, 2014

Solve the following problems. Please remember to use complete sentences and good grammar. Four points each.

1. Show that there are the same number of primitive roots modulo $2p^t$ as there are modulo p^t, where p is an odd prime and t is a positive integer.
2. Show that the integer m has a primitive root if and only if the only solutions of the congruence $x^2 \equiv 1 \pmod{m}$ are $x \equiv \pm 1 \pmod{m}$.
3. Let p be an odd prime.
   (a) Show that if p is an odd prime an r is a primitive roots of p, then $ind_r(p-1) = (p-1)/2$.
   (b) Show that the congruence $x^4 \equiv -1 \pmod{p}$ has a solution if and only if p is of the form $8k+1$.
4. Find all solutions of the following congruence: $13^x \equiv 5 \pmod{23}$.
5. Let p be an odd prime and $p \not| a$. Show that there exist integers u, v with $(u, v) = 1$ so that $u^2 + av^2 \equiv 0 \pmod{p}$ if and only if $-a$ is a quadratic residue modulo p.
6. Show that if p is a prime and $p = 8n+1$, and r is a primitive root modulo p, then the solutions of $x^2 \equiv \pm 2 \pmod{p}$ are given by $x \equiv \pm(r^{7n} \pm r^n) \pmod{p}$, where $\pm$ sign in the first congruence corresponds to the $\pm$ sign inside the parentheses in the second congruence.
7. (Bonus) A *cyclic number* is an $(n-1)$-digit integer that, when multiplied by $1, 2, 3, \ldots, n-1$, produces the same digits in a different order. For example, 142857 is a cyclic number with 6 digits. Prove that if 10 is a primitive root modulo p, where p is a prime, then $(10^{p-1} - 1)/p$ is a cyclic number.