

Math 432 Homework Five

Due: Friday, Feb 26, 2016

- (1) Find the number of integers between 1 and 10000 that are not divisible by 4, 6, 7, or 10.
- (2) Count the permutations $i_1 i_2 i_3 i_4 i_5 i_6$ of $[6]$, where $i_1 \neq 1, 5$; $i_3 \neq 2, 3, 5$; $i_4 \neq 4$ and $i_6 \neq 5, 6$.
- (3) Give a combinatorial proof (via inclusion-exclusion) for the identity below.

$$\sum_{k=0}^n (-1)^k \binom{n}{k} \binom{2n-2k}{n-1} = 0.$$

- (4) Let $S(n, k)$ be the number of ways to partition $[n]$ into k nonempty subsets ($S(n, k)$ is called the Stirling number of second kind). Prove the following identity by inclusion-exclusion:

$$S(n, k) = \frac{1}{k!} \sum_{i=0}^k (-1)^i \binom{k}{i} (k-i)^n$$

- (5) Let a_n be the number of words of length n from the alphabet $\{w, x, y, z\}$ such that x appears an even number of times and y appears an odd number of times. Build the EGF for the sequence $a_1, a_2, \dots, a_n, \dots$ and use it to obtain a formula for a_n .
- (6) Let a_n be the number of involutions on an n -element set. Derive a recurrence for the number, and use the generating function method to obtain the EGF from it.