Math 432 lec 21 edge coloring and Hamiltonian cycle

1. Edge coloring

- Def: k-edge-coloring. chromatic index.

- Line graph $L(G)$, and $\chi'(G) = \chi(L(G))$.

- Ex: chromatic index of K_{2n}, K_{2n+1}, Petersen graph,

- $\chi'(G) \geq \Delta$, and equality holds for bipartite graphs.

- Vizing's Thm: $\Delta \leq \chi'(G) \leq \Delta + 1$. (pf: downshift and Kemp Chain)
  A graph is in Class I if $\chi'(G) = \Delta$ and in Class II if $\chi'(G) = \Delta + 1$.

2. Hamiltonian cycle

- Def: Hamiltonian cycle and Hamiltonian graph
- Sufficient conditions:
  - Dirac Thm: $\delta(G) \geq n/2$, then G is Hamiltonian. The degree is sharp.

  - Ore Thm: if $d(u) + d(v) \geq n$ for every $uv \notin E(G)$, then G is Hamiltonian.

  - Thm: if $\kappa(G) \geq \alpha(G)$ and $G \neq K_2$, then G is Hamiltonian. (pf: take a longest cycle and then find the neighbors of the cycle on a component, the ones next to the attachment plus one vertex from the component is (too large) independent set.)