

MATH 214 SAMPLE HOMEWORK

Solve the following problems. Due Feb 5, 2009.

- (1) Recall the Knights and Knaves from the first day of class. Recall that knights always tell the truth, and knaves always lie. You meet three inhabitants: Patricia, Quinn and Roberta. Patricia claims that it's false that Roberta is a knave. Quinn says, 'Either Roberta is a knight or I am a knight.' Roberta says that Quinn is a knave. Who are knights and who are knaves? Prove your answer (using truth table).
- (2) Later, you meet Ann, Bert and Chuck. Ann says, 'Chuck could claim that I am a knight.' Bert says that only a knave would say that Ann is a knave. Chuck claims, 'Ann could say that I am a knave.' Who are knights and who are knaves? Prove your answer.
- (3) By using truth tables prove that, for all statements P and Q, the statement ' $P \Rightarrow Q$ ' and ' $(\text{not } Q) \Rightarrow (\text{not } P)$ ' are equivalent.

- (4) Prove that for all real numbers a, b and c ,

$$bc + ac + ab \leq a^2 + b^2 + c^2.$$

- (5) Prove that for all real numbers a and b ,

$$|a| < |b| \Rightarrow a^2 \leq b^2.$$

- (6) Prove that in a graph the number of vertices with odd degrees is even.