Math 412 Homework 2

your name

Due date: Sept 10, 2010

Solve the following problems. Please remember to use complete sentences and good grammar. Each problem is 4 points.

1. Show that $8a + 3$ and $5a + 2$ are coprime for all integers a.

2. Show that there are infinite many primes of the form $3k + 2$.

3. Let $S = \{\log_{10} p : p \text{ prime}\}$. Prove that the elements of S are linearly independent over Q, the rationals.

4. Show that $(ac, bc) = c(a, b)$ for any integers a, b, c.

5. Show that $\sqrt{2} + \sqrt{3}$ and $\log_{10} 5432$ are irrational.

6. Show that $11 | a^2 + 5b^2$ if and only if $11|a$ and $11|b$ for $a, b \in Z$.