Math 412 Homework 9

your name

Due date: Dec 3, 2010

Solve the following problems. Please remember to use complete sentences and good grammar.

1. Solve the Pell equation: $x^2 - 29y^2 = 1$.

2. Show that the Gaussian integers α and β are associate if $\alpha|\beta$ and $\beta|\alpha$.

3. Find all Gaussian integers α, β, γ such that $\alpha\beta\gamma = \alpha + \beta + \gamma = 1$.

4. Let $x, y \in \mathbb{Z}$. Show that if $(x, y) = 1$, then x and y are also coprime in $\mathbb{Z}[i]$.

5. Let $\alpha, \beta \in \mathbb{Z}[i]$ and are coprime. If $\alpha\beta = \gamma^2$ for some $\gamma \in \mathbb{Z}[i]$, then α and β can also be written as square, up to a factor of units.

6. show that if x and y are integers such that $x^2 + 1 = y^3$, then $x - i$ and $x + i$ are relatively prime.