Math 432 – Combinatorics
Homework 1
Due Friday Jan. 29 midnight, 2016.

Solve the following problems. Show all your work. Please type your homework by way of Latex and submit your .pdf file through blackboard. Each problem is 4 points if not specified.

1. (6 points) In terms of binomial coefficients, count the (5-card) poker hands that have
   a) three-of-a-kind (three cards of the same rank and one in each of two other ranks).
   b) full house (two cards of equal rank and three cards of another rank).
   c) straight flush (five cards in sequence from the same suite).

2. Prove the following identity
$$\sum_{k=0}^{n} k\binom{n}{k} = n2^{n-1}.$$

3. Prove the following identity:
$$\sum_{i=0}^{k} \binom{m}{i}\binom{n}{k-i} = \binom{m+n}{k}.$$

4. By calculating the coefficient of x^n on the two sides of the identity $(1+x)^n \cdot (1-x)^n = (1-x^2)^n$, prove that
$$\sum_{k=0}^{n} (-1)^k \binom{n}{k}^2 = \begin{cases} 0, \text{ if } n \text{ is odd} \\ (-1)^m\binom{2m}{m}, \text{if } n = 2m. \end{cases}$$

5. Compute $\sum_{k=0}^{n} k^2\binom{n}{k}$.