Math 432 Homework Ten

Due: Friday, April 8, 2016

Prove the following statements. Four points for each.

(1) Show that $\chi(G) = \max\{\chi(B) : B \text{ a block of } G\}$.

(2) (6 points) A graph is k-degenerate if it can be reduced to K_1 by repeatedly deleting vertices of degree at most k.
(a) Show that a graph is k-degenerate if and only if every subgraph has a vertex of degree at most k.
(b) Characterize the 1-degenerate graphs.
(c) Show that every k-degenerate graph is $(k+1)$-colorable.

(3) Let $\overline{G}$ be the complement graph of graph G, that is, an edge in $\overline{G}$ if and only if it is not in G. Show that $\chi(G) + \chi(\overline{G}) \leq n(G) + 1$.

(4) Let G be a graph whose odd cycles are pairwise intersecting, meaning that every two odd cycles in G have a common vertex. Prove that $\chi(G) \leq 5$. Construct a graph to show that the bound cannon be improved.

(5) Prove that if G has no induced $2K_2$, then $\chi(G) \leq \binom{\omega(G)+1}{2}$.

(6) For all $k \in \mathbf{N}$, prove that a graph G is 2^k-colorable if and only if G can be decomposed to k edge-disjoint bipartite graphs.