

Math 432 Homework Eleven

Due: Friday, April 15, 2016

Prove the following statements. Four points for each.

- (1) Prove that every regular graph having a cut-vertex is Class 2 (i.e., the chromatic index is $\Delta + 1$).
- (2) Let G be a simple graph.
 - (a) Prove that the number of edges in $L(G)$ is $\sum_{v \in V(G)} \binom{d(v)}{2}$.
 - (b) Prove that G is isomorphic to $L(G)$ if and only if G is 2-regular.
- (3) Let G be a bipartite graph with maximum degree k . Let f be a proper k -edge-coloring of a subgraph H of G . Let uv be an edge not in H . By using a path alternating in two colors, show that f can be altered and then extended to a proper k -edge-coloring of $H + uv$. Conclude that $\chi'(G) = \Delta(G)$.
- (4) Prove that every 2-connected simple graph G has a cycle of length at least $\min\{n(G), 2\delta(G)\}$. Use this to prove that every $2k$ -regular graph with $4k + 1$ vertices is Hamiltonian.
- (5) Let $R(p, q)$ be the Ramsey number. Prove that $R(p, q) \leq R(p-1, q) + R(p, q-1)$.
- (6) Find $R(K_3, 2K_2)$.