

Math 432 – Combinatorics

Homework 3

Due: Feb 12 11:58pm, 2016.

Work the following problems. Show all your work. Four points each if not otherwise specified.

1. (6 points) Let a_n be the number of n -tuples (a_1, a_2, \dots, a_n) with $a_i \in [4]$ that have at least one 1 and have no 2 appearing before the first 1.
 - Obtain and solve a recurrence for a_n .
 - Give a direct counting argument (without using summations) to prove the resulting simple formula.
2. Solve the following recurrence relation using characteristic equation method.
 $h_n = 4h_{n-1} + 8h_{n-2} + 3n$, ($n \geq 3$), with $h_1 = h_2 = 1$.
3. Solve the following recurrence relation using generating function method:
 $a_n = 5a_{n-1} - 6a_{n-2} + 2^n$ for $n \geq 3$, with $a_1 = a_2 = 1$.
4. (6 points) Let a_n be the number of words of length n on the alphabet $\{0, 1, 2\}$ such that 1 and 2 are never adjacent.
 - Obtain a recurrence relation for a_n .
 - Solve for a_n using both the characteristic equation method and the generating function method.
5. (6 points) Let $a_{n,k}$ denote the number of ways to partition n people in a row into k groups so that no two consecutive people are in the same group.
 - Obtain a recurrence relation for $a_{n,k}$.
 - Let $A_k(x) = \sum_{n=0}^{\infty} a_{n,k}x^n$. Use the generating function method to express $F_k(x)$ as a ratio of polynomials.