

Math 432 Homework Six

Due: Friday March 4, 2016

Prove the following statements. Four points for each.

- (1) A cycle is a circuit without repeated vertices. Prove that every n -vertex graph with at least n edges contains a cycle.
- (2) Prove or disprove: if G is an Eulerian graph with edges e, f that share a vertex, then G has an Eulerian circuit in which e, f appear consecutively.
- (3) A list of integers d_1, d_2, \dots, d_n with $d_1 \geq d_2 \geq \dots \geq d_n > 0$ is graphic if it is the degree list of some simple graph. Show that
 - (a) $(7, 6, 5, 4, 3, 3, 2)$ and $(6, 6, 5, 4, 3, 3, 1)$ are not graphic.
 - (b) if a sequence is graphic, then $\sum_{i=1}^n d_i$ is even and $\sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min\{k, d_i\}$ for $1 \leq k \leq n$.
 - (c) (bonus problem) the condition in (b) is also sufficient.
- (4) Show that the Kruskal's algorithm produces a minimum spanning tree in a weighted connected graph.
- (5) We showed in class that every tree is a bipartite graph. Prove that every tree has a leaf in its larger partite set (in both if they have equal size).