Math 432 Homework Seven

Due: Saturday, March 19, 2016

Prove the following statements. Four points for each.

(1) Prove that Petersen graph has no cycle of length 7 or 10.

(2) Let G be the 3-regular graph with $4m$ vertices formed from m pairwise disjoint kites (K_4 minus an edge) by adding m edges to link them in a ring. Prove that it has $2m \cdot 8^m$ spanning trees.

(3) (6 points) Count the following sets of trees with vertex set $[n] = \{1, 2, \ldots, n\}$, given two proofs for each: one using the Prufer correspondence and one by direct counting arguments.
(a) trees that have 2 leaves.
(b) trees that have $n - 2$ leaves.

(4) Prove that a d-regular simple graph G has a decomposition into copies of $K_{1,d}$ if and only if it is bipartite.

(5) Prove that a graph is bipartite if and only if every subgraph H of G has an independent set consisting of at least half of $V(H)$, where a set of vertices is independent if there is no edge between the vertices in the set.

(6) Let $A = (A_1, \ldots, A_m)$ be a collection of subsets of a set Y. A system of distinct representatives (SDR) for A is a set of distinct elements $a_1, a_2, \ldots, a_m$ in Y such that $a_i \in A_i$. Prove that A has an SDR if and only if $|\cup_{i \in S} A_i| \geq |S|$ for every $S \subseteq \{1, 2, \ldots, m\}$.