Math 432 Homework Eight

Due: Friday, March 25, 2010

Prove the following statements. Four points for each.

(1) The defect $df(S)$ of a set $S \subseteq X$ in an X, Y-bigraph is defined to be $|S|-|N(S)|$. The matching number $\alpha'(G)$ is defined to be the maximum size of a matching in G. Prove that in an X, Y-bigraph G, $\alpha'(G) = |X| - \max_{S\subseteq X}\{df(S)\}$.

(2) Use the Konig-Egervary Theorem to prove that every subgraph of $K_{n,n}$ with more than $(k-1)n$ edges has a matching of size at least k.

(3) Suppose that G is an r-connected graph of even order having no $K_{1,r+1}$ as an induced subgraph. Prove that G has a perfect matching.

(4) Prove that $\kappa'(G) = \kappa(G)$ if G is a 3-regular simple graph. Find (with proof) the smallest 3-regular simple graph having connectivity 1.

(5) Let H be the block-cutpoint graph of a graph G that has a cut-vertex.
(a) prove that H is a forest.
(b) Prove that G has at least two blocks that contain one cut-vertex of G.
(c) Prove that every graph has fewer cut-vertices than blocks.

(6) Suppose that G has no isolated vertices. Prove that if G has no even cycles, then every block of G is an edge or an odd cycle.