Math 432 Homework Nine

Due: Saturday, April 2, 2016

Prove the following statements. Four points for each.

(1) Prove that if G is 2-connected, then $G - xy$ is 2-connected if and only if x and y lie on a cycle in $G - xy$. Conclude that a 2-connected graph is minimally 2-connected if and only if every cycle is an induced subgraph.

(2) Let G be a k-connected graph, and let S, T be disjoint subsets of $V(G)$ with size at least k. Prove that G has k pairwise disjoint S, T-paths.

(3) Suppose that S is a set of k vertices in a k-connected graph G, with $k \geq 2$. Prove that G has a cycle containing S.

(4) Prove that a set of edges in a connected plane graph G forms a spanning tree of G if and only if the duals of the remaining edges form a spanning tree of G^*.

(5) Use Euler's formula to count the regions formed by n pairwise-crossing lines in the plane, where no three lines have a common point. (Comment: proving the formula by induction is not acceptable. Hint: modify the picture to obtain a finite plane graph.)

(6) Give two proofs that the Petersen graph is nonplanar.
a) Using Kuratowski's Theorem
b) Using Euler's Formula and the fact that the Petersen graph has girth 5.

(7) Use Euler's Formula and properties of planar graphs, prove that for any planar graph G,

$$\sum_{v \in V(G)} (d(v) - 4) + \sum_{F \in F(G)} (d(F) - 4) = -8,$$

where $F(G)$ is the set of faces of G.