Math 432 lec 18 Planar graphs

Def: curve, polygonal curve, drawing, crossing. Planar graph (A graph is planar if it can be drawn on the plane so that no crossing edges), plane graph (a drawing of a planar graph on a plane is called a plane graph). open set, region and faces.

Jordan Curve Theorem: A simple closed polygonal curve C consisting of finitely many segments partitions the plane into exactly two faces, eating having C as boundary.

Thm: K_5 and $K_{3,3}$ cannot be drawn on the plane without crossings. (pf: a separating cycle with two many chords)

Thm (Kuratowski 1930) a graph is planar iff it does not contain a subdivision of K_5 or $K_{3,3}$.

Def: Dual graph G^*. face degree.

Prop: $\sum_F d(F) = 2e(G)$.

Euler formula: connected plane graph has $n - e + f = 2$. (pf: induction on e)

Thm: if G is a simple planar graph with $n \geq 3$, then $e(G) \leq 3n - 6$; if G has girth g (the length of the shortest cycle), then $e(G) \leq \frac{g}{g-2}(n-2)$; in particular, if G is bipartite, then $e(G) \leq 2n(G) - 4$.

Application of Euler formula on Regular polyhedra (Platonic solids): tetrahedron (4-face), cube(6-face), octahedron(8-face), dodecahedron(12-face), icosahedron (20-face)