

Math 214 – Foundations of Mathematics
Homework 1

Due: noon September 7, 2012.

Solve the following problems. Please remember to use complete sentences and good grammar. Each problem is 4 points.

1. Write each of the following sets as specified.
 - (a) List the elements in the set $A = \{n \in \mathbb{N} : n^3 < 100\}$.
 - (b) Describe the set $B = \{-3, -2, -1, 0, 1, 2, 3\}$ using the notation $\{n : p(n)\}$, where $p(n)$ specifies the property of element n .
2. Recall that for a set A , $\mathcal{P}(A)$ denotes the power set of A .
 - (a) Find $\mathcal{P}(\mathcal{P}(\{1\}))$ and its cardinality.
 - (b) Give examples of a set S such that $S \subseteq \mathcal{P}(\mathbb{N})$ and $|S| = 5$.
 - (c) Give examples of a set S such that $S \in \mathcal{P}(\mathbb{N})$ and $|S| = 5$.
3. The following problems involve set operations.
 - (a) Given an example of three sets A, B , and C such that $B \neq C$ but $B - A = C - A$.
 - (b) Let $A = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}$. Find $\{\emptyset, \{\emptyset\}\} \cap A$.
4. For a real number r , define S_r to be the interval $[r - 1, r + 2]$. Let $A = \{1, 3, 4\}$. Determine $\cup_{\alpha \in A} S_\alpha$ and $\cap_{\alpha \in A} S_\alpha$.
5. For two sets A and B , recall that $A \times B$ is the Cartesian product of A and B .
 - (a) Let $A = \{a, b\}$. Determine $A \times \mathcal{P}(A)$.
 - (b) Let $A = \{0, 1\}$ and $B = [0, 2] \cap [1, 3]$. Depict the set $A \times B$ in \mathbb{R}^2 .
 - (c) Let $A = \{0, 1\}$, $B = (0, 1) \cap A$ and $C = \mathbb{R}$. What is $A \times B \times C$?
6. Determine all different partitions of the set $\{1, 2, 3\}$.
7. Let P : 15 is odd, Q : 21 is prime and R : $\frac{1}{2} \in \mathbb{N}$. State each of the following in words, and determine whether they are true or false.
 - (a) $P \vee Q$
 - (b) $P \wedge Q$
 - (c) $(\sim P) \vee Q$
 - (d) $P \wedge (\sim Q \vee R)$