Math 214 – Foundations of Mathematics
Homework 11

Due noon Dec 4, 2012

Each problem worths 4 points unless specified otherwise.

1. Let $\emptyset \neq I \subseteq \mathbb{N}$. For each $i \in I$, A_i is denumerable. Show that $\cup_{i \in I} A_i$ is denumerable.
2. Let $A = \{(\alpha_1, \alpha_2, \alpha_3, \ldots) : \alpha_i \in \{0, 1\}, i \in \mathbb{N}\}$, i.e., A is the infinite cartesian product of the set $\{0, 1\}$. Show that A is uncountable.
3. Prove that the intervals $[0, \infty)$ and $(-1, 4)$ have the same cardinality.
4. (6 points) Determine the cardinality of the following sets (finite, denumerable, or uncountable), and justify your answers:
   (a) the set of all open intervals with rational midpoints.
   (b) the set of all open intervals with rational endpoints.
5. Show that $|A| < |\mathbb{N}|$ for every finite set A.
6. Use Schröder-Bernstein Theorem to prove that $|(2, 5)| = |(0, 1]|$.