

Math 214 Homework 2

Your name

Due: noon Sept 14, 2012

Solve the following problems. Please remember to use complete sentences and good grammar.

- (4 points) In each of the following, two open sentences $P(x)$ and $Q(x)$ over a domain S are given. For each part, determine $T = \{x \in S : P(x) \Rightarrow Q(x)\}$.
 - $P(x) : x - 3 = 4; Q(x) : x \geq 8; S = \mathbb{R}$.
 - $P(x) : x \in [-1, 2]; Q(x) : x^2 \leq 2; S = [-1, 1]$.
- (4 points) Let P, Q be statements. Show that $\sim (P \implies Q)$ and $P \wedge (\sim Q)$ are logically equivalent using truth table.
- (6 points) Write the statements so that there are no \sim symbols. Then, rewrite the statements so that there are no \forall, \exists, \in or $=$ symbols.
 - $\sim (\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, xy = 1)$;
 - $\sim (\exists y \in \mathbb{R}, \forall x \in \mathbb{R}, xy = 0)$;
 - $\sim (\exists n \in \mathbb{Z}, \exists m \in \mathbb{Z}, m \leq n)$;
- (2 points) Consider the sentence, "For every integer $n > 0$ there is some real number $x > 0$ such that $x < 1/n$." Without using words of negation, write a complete sentence that negates the sentence. Which sentence is true?
- (4 points) For $\alpha \in \mathbb{R}$, let $S_\alpha = (-\alpha, \alpha)$. Prove or disprove the following statements.
 - $\forall \alpha \in (0, 1), \exists \beta \in (0, 1), S_\alpha \subset S_\beta$ (note that \subset and \subseteq are not the same).
 - $\exists \alpha \in (0, 1), \forall \beta \in (0, 1), S_\alpha \subset S_\beta$.
- (4 points) Let A, B, C be sets. Prove that $(A - B) \cup (A - C) = A - (B \cap C)$.
- (4 points) Let A, B, C and D be sets. Prove that

$$(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D).$$