

Math 214 – Foundations of Mathematics  
Homework 4

Due Sept 28, 2012

Your name

Solve the following problems. Show all your work. Every problem worths 4 points.

1. Prove that  $\sum_{k=1}^n \frac{1}{(k+2)(k+3)} = \frac{n}{3n+9}$  for every positive integer  $n$ .
2. Prove that  $(2012)! > 2^{2012}$ . (Hint: prove a general statement in terms of  $n$ .)
3. For each of the following sets, determine whether it is well-ordered and show your reasons.
  - (a)  $S = \{n \in \mathbb{N} : n \text{ is even}\}$ .
  - (b)  $T = \{n \in \mathbb{Z} : n \text{ is odd}\}$ .
4. A sequence  $\{a_n\}$  is defined recursively by  $a_1 = 1, a_2 = 4, a_3 = 9$ , and

$$a_n = a_{n-1} - a_{n-2} + a_{n-3} + 2(2n - 3)$$

for  $n \geq 4$ . Conjecture a formula for  $a_n$  and prove that your conjecture is correct.

5. Use Strong Principle of Mathematical Induction to prove that for each integer  $n \geq 10$ , there are non-negative integers  $a$  and  $b$  such that  $n = 2a + 7b$ .
6. Prove that  $7|(3^{2n} - 2^n)$  for every nonnegative integer  $n$ .