

Math 214 – Foundations of Mathematics  
Homework 5

**Due noon Oct 5, 2012**

Your name

Solve the following problems. Show all your work. Every problem worths 4 points.

1. Let  $a, b, c, d \in \mathbb{Z}$  with  $a, c \neq 0$ . Prove that if  $a|b$  and  $c|d$ , then  $ac|(ad + bc)$ .
2. Prove that for integers  $n$  and  $m$ ,  $3|nm$  if and only if  $3|m$  or  $3|n$ . (Hint: in one direction, you may want to use Division Theorem to write  $m, n$  in some special form)
3. Find all primes,  $p$ , that can be written as  $p = n^3 - 1$  for some  $n \in \mathbb{N}$ . (Make sure that you have found all of them.)
4. Prove or disprove: if  $a$  and  $b$  are odd integers, then  $4|(a - b)$  or  $4|(a + b)$ .
5. Prove that for any  $n \in \mathbb{Z}$ ,  $n^2$  cannot be of the form  $3m - 1$ , where  $m \in \mathbb{Z}$ .
6. Show that an positive integer is a multiple of 9 if and only if the sum of all digits of the integer is a multiple of 9.
7. Find  $\gcd(51, 288)$  and  $m, n \in \mathbb{Z}$  such that  $\gcd(51, 288) = 51n + 288m$  (Show your intermediate quotients and remainders).