Math 214 – Foundations of Mathematics
Homework 8

Due Nov 2, 2011

Your name

Solve the following problems. Show all your work. Four points each.

1. (4 points) Let S be a nonempty subset of $\mathbb{Z}$, and let R be a relation defined on S by $(x, y) \in R$ if $3|(x + 2y)$.
   (a) Prove that R is an equivalence relation.
   (b) If $S = \{-7, -6, -2, 0, 1, 4, 5, 7\}$, then what are the distinct equivalence classes in this case?

2. (4 points) Let S be a non-empty subset of $\mathbb{N}$, and let $\sim$ be a relation defined on S by $x \sim y$ if $x^2 + y^2$ is even. Prove that $\sim$ is an equivalence relation. Determine the distinct equivalence classes.

3. (4 points) Show that the relation R defined on $\mathbb{R} \times \mathbb{R}$ by $((a, b), (c, d)) \in \mathbb{R}$ if $|a| + |b| = |c| + |d|$ is an equivalence relation. describe geometrically the elements of the equivalence classes $[(1, 2)], [(3, 0)]$.

4. (4 points) For some nonempty set S, suppose $f : S \to S$ is a function and an equivalence relation. What is f? Justify your answer.

5. (4 points) Define the mapping $h : \mathbb{Z}_{20} \to \mathbb{Z}_{20}$ by $h([a]) = [3a]$ for each $a \in \mathbb{Z}$. Prove that h is well-defined and injective.

6. (4 points) Let p be a positive prime number and $f : \mathbb{Z}_p \to \mathbb{Z}_p$ be defined as $f([x]) = [x^2]$. Show that f is a function. Give examples to show that it is not necessarily injective or surjective.