Math 214 – Foundations of Mathematics
Homework 9

Due Nov 9, 2012

your name

Solve the following problems. Show all your work. Unless otherwise stated, problems are worth 4 points.

1. Consider $h : \mathbb{Z}_{16} \to \mathbb{Z}_{24}$ by $h([a]) = [3a]$ for each $a \in \mathbb{Z}$.
   (a) Prove that h is a function.
   (b) Is h injective? Surjective? Bijective?

2. Let $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R} \times \mathbb{R}$ where, for $(a, b) \in \mathbb{R}$, $f(a, b) = (2a + 7, 3b - 3)$. Prove that f is a bijective function and find f^{-1}.

3. Let A, B and C be nonempty sets and let f, g and h be functions such that $f : A \to B$, $g : B \to C$ and $h : B \to C$. For each of the following,prove or disprove:
   (a) if $g \circ f = h \circ f$, then $g = h$.
   (b) if f is injective and $g \circ f = h \circ f$, then $g = h$.

4. For nonempty sets A, B, C, let $f : A \to B$ and $g : B \to C$ be functions.
   (a) Prove that if $g \circ f$ is injective, then f is injective.
   (b) Disprove that if $g \circ f$ is injective, then g is injective.

5. For nonempty sets A and B and functions $f : A \to B$ and $g : B \to A$ suppose that $g \circ f = i_A$, the identity function on A.

   (a) *(4 Points)* Show that f is injective and g is surjective.

   (b) *(2 Points)* Show that f is not necessarily surjective.

   (c) *(2 Points)* Show that g is not necessarily injective.

6. Let $A_1, A_2 \subseteq A$. Prove that if f is injective, then $f(A_1) \cap f(A_2) \subseteq f(A_1 \cap A_2)$. Give an example that the equality fails.

7. (Extra Credit, 4 points) Le S be the set of odd positive integers. A function $F : \mathbb{N} \to S$ is defined by $F(n) = k$ for each $n \in \mathbb{N}$, where k is that odd positive integer for which $3n + 1 = 2^m k$ for some nonnegative integer m. Prove or disprove the following:
   (a) F is injective.
   (b) F is surjective.